

"All men by nature, desire to know."¹ (tllroughtout history the need to know has been a prime source of governing mens' actions. This need has founded civilizations, it has started wars, and it has led man to his ultimate control of his environment. I shall examine the causes and developments of mathematics. Starting with early Egypt and Babylon, then on to classical Greece, and finally the 17th century through modern times; I will trace the need and development of mathematics.

"Priority in the development of mathematics belongs to Babylon, where ancient land numeration, algebra, and geometry methods existed at least from the Hammurabi dynasty, around 1700 B. C.,² although "Neither Egypt nor Rome advanced beyond the level of elementary practical arithmetic and mensuration." These ancient civilizations developed mostly practical mathematics, but thier effect, even upon modern mathematics, is still enormous. I shall cover both the need and the methods used in this ancient mathematics.

"There is little doubt that mathematics arose from necessity. The annual flooding of the Nile valley, for example, forced the ancient Egyptians to develop some

1 Aristotle

2 "Mathematics, History", Encyclopedia Britannica, volume 11, page 642

J Ibid

system of reestablishing land boundaries.,,4 Increased barter increased the need for early practical arithmetic.⁵ The need for a calendar, if a basic one, led to development in mathematics; "the astronomy of the old Babylonian period was just adequate for maintaining the calendar, on which the irrigation system supporting the civilization depended.,,6 Civilization and mathematics are inseparable i. e. "Mathematics beyond primitive counting originated with the evolution of advanced forms...-society."⁷ As Aristotle once pointed out; a civilization is necessary "to separate a thinking class from the working class.

Early mathematics consisted almost exclusively of trial and error. Early Egyptian mathematics was geometry.⁸ The Egyptians also developed formulas for the areas and volumes of many shapes, but used trial and error rather than proofs, so they were not entirely correct in their formulas.⁹ The Babylonians were only more advanced than the Egyptians. "The Babylonians were interested in number relations beyond the merely practical mathematics.,,10 i. e. "An old Babylonian text (1700 B. C.) investigates

4 "Mathematics", Encyclopedia Americana, volume 17, page 392
 5 Ibid
 6 Michal Moffatt, The ages of Mathematics vol. 1, Page 35
 7 "Mathematics",Encyclopedia Americana, volume 17, Page 388
 8 The word geometry is from a Greek word meaning "measure of the land".
 9 Michal, Moffatt, The ages of Mathematics vol. 1, page 43
 10 Ibid

triples of Pythagorean numbers; that is, numbers satisfying the relationship $a^2 + b^2 = c^2$.¹¹ "The facility of numerical computation, as the result of the place value notation, is everywhere visible in Babylonian mathematics."¹² The Babylonians had a sort of combination between base 10 and base 60 with a zero.¹³

The accomplishments of ancient Greece dwarfed those of Egypt and Babylon, and rival the achievements of the 17th and 18th centuries! It was during this period that such great men as Pythagoras, Euclid, and Archimedes lived. "The Greeks insisted that mathematical facts must be established not by empirical procedures but by deductive reasoning."¹⁴ This led to a system of undefined terms, axioms, theorems et cetera.¹⁵ Although most Greek mathematicians; "Purely algebraic notation had been

11 "Mathematics, History", Encyclopedia Britannica, volume 11, page 644

12 Ibid

13 Bases are a part of the place value notation, that is the position of each digit tells its magnitude. For example, the number 94071 means $9 \times 10^4 + 4 \times 10^3 + 0 \times 10^2 + 7 \times 10^1 + 1 \times 10^0$. If we were to write a number in another base besides base 10 it would be like: 13702 (base 8) equals $1 \times 8^4 + 3 \times 8^3 + 7 \times 8^2 + 0 \times 8^1 + 2 \times 8^0$, of B3AI06 (base 12) = $B0(11) \times 12^5 + 3 \times 12^4 + A(10) \times 12^3 + 1 \times 12^2 + 6 \times 12^1$. All bases need the number of different characters equal to the number of the base (zero is one of the digits). Without zero a place value system cannot exist, the number 57,302, for example, would have to be written 5732, which could mean 500,732 or 5,732,000 or just 5732 or any number of other things.

14 "Mathematics", Encyclopedia Americana, volume 17, Page 393

15 Michal Moffatt, The ages of Mathematics vol. 1, page 67

used by Aristotle in his investigation of formal logic, 1116
and the concept of area gave rise to a near rigorous theory
of integration¹⁷ even in ancient times. ills

The first great thinker in Greek history was Thales of
Miletus.¹¹¹⁹ Thales as a youth went to Egypt to learn the methods
from the priests. He was soon surpassing their methods, which
they established by trial and error and held in mystic regard.
After he had learned all their knowledge, he went back to Greece
and set up a school. In his school he set up a series of propo-
sitions (axioms) and derived things with deductive methods. ~ll' rth ~. -l, j, r
Thales lived from 567 B. C. to 497 B. C. \.._21/He was one of /jJp4'fC,
the seven Wise men of Greece, the only one who didn't become ~, /, "/ if
one due to politics.²² /\$

The next major Greek mathematician was Pythagoras. Pythag-
oras was a student of Thales, and like Thales he went to Egypt.
Pythagoras formed a school in Croton and let everyone, even
women, learn there^V for free!²³ He soon furthered the work on
a deductive system started by Thales, building theorems upon
theorems.

Pythagoras made five propositions which he 'proved; from

11 Mathematics, History, Encyclopedia Britannica,
VOLUME 11, page 647

17 Integration is a function of calculus.

18 Mathematics, History, Encyclopedia Britannica,
volume 11, page 647

19 Michal Moffatt, The ages of Mathematics vol 1,
page 73

20 Leon Perry, The Mathmen, page 17

21 Ibid

22 Michal Moffatt, The ages of Mathematics vol 1, page 82

known 'facts'.

- 1) The sum of the angles of a triangle equals two right angles (180°).
- 2) The sum of the exterior angles of a triangle equals four right angles (360°).
- 3) The sum of the interior angles of a polygon equals $2n-4$ right angles, where n is the number of sides.
- 4) The sum of the exterior angles of any polygon equals four right angles (360°), regardless of the number of sides.
- 5) Three regular polygons - a triangle, a square, and a hexagon - fill the space about a point on a plane.

There is some question about the validity of these proofs, however. Not all of the 'facts' he assumed to prove them are valid. Non-Euclidean geometry, which is consistent and actually better describes Einsteinian spaces based on the assumption that his 'facts' are false (This discussion is essentially the same) as that of the parallel postulate²⁵ which I shall discuss much later in this paper.

Any discussion of Pythagoras must include his remarkable theories of numbers.

Pythagoras believed that all things - physical and mental, all nature and all ideas - are built on a pattern of integers. Fractions he did not consider numbers. They were only ratios, relations between numbers. Having discovered the figurate numbers,

24 George Gamow, One, Two, Three, Infinity, page 103

25 Pythagoras's proposition number one and Euclid's parallel postulate can each be proven from the other.

Pythagoras thought that he proved that geometry itself was formed on some sort of number pattern. And it followed 'logically', he asserted, that all material things in nature were formed in a similar manner and that numbers rule the universe.~j

Pythagoras discovered prime and composite numbers. Another type of number that he discovered are 'perfect' numbers. Perfect numbers are numbers that equal the sum of their factors. Six, for example, equals one plus two plus three. Pythagoras discovered the perfect numbers 6 and 28, his students discovered 496 and 8128. It was another 1500 years before the next perfect number, 3,550,336, was discovered. Today 17 are known, the highest of which is over 1300 digits long!27 'Friendly' numbers are also an invention of Pythagoras. They are numbers which are the sums of each others factors i. e. $284 = 1 + 2 + 4 + 5 + 10 + 11 + 20 + 22 + 44 + 55 + 110$ (the factors of 220), $220 = 1 + 2 + 4 + 71 + 142$ (the factors of 284). The above numbers are the only friendly numbers that Pythagoras discovered! It was 2000 years before another pair was discovered! Leonard Euler alone found 60 pairs and today over 400 pairs are known.

Still another group of numbers are male and female numbers~ One is the ancestor of numbers, not a number. Even numbers are female. Odd numbers are male. Five is the marriage number

26 Leon Perry, The Mathmen, page 32

87 Ibid

because it is the sum of the first male number and the first female number (2 + 3). Figurate numbers are divided into triangular numbers, square numbers, pentagonal numbers et cetera. They correspond to patterns of dots in triangles, squares, pentagons et cetera. Triangular numbers are obtained by adding 2 then 3 then 4 et cetera to the number 1 i. e. $1 + 2 = 3$, $3 + 3 = 6$, $6 + 4 = 10$, $10 + 5 = 15$ et cetera. This corresponds to triangles of 2 on a side, 3 on a side, 4 on a side et cetera. Square numbers correspond to squares of 2 wide, 3 wide, 4 wide et cetera; which can be obtained by adding 3 then 6 then 7 et cetera to the number 1. The other are similar. Pythagorean triples are what Pythagoras is most famous for. They are integers which fulfill the condition $a^2 + b^2 = c^2$, and a, b, and c will be the lengths of the sides of a right triangle. Pythagoras developed a formula to figure out such numbers:

$$n^2 + ((n^2 - 1)/2)^2 = ((n^2 + 1)/2)^2$$

where n is an odd integer.

"One day Pythagoras discovered what was to him an incredible fact: there were numbers which were neither integers nor fractions!"²⁸ He was making an isosceles right triangle and found the hypotenuse to have a length of $\sqrt{2}$: which he found no integer nor fraction for. He, however, could not prove it was irrational. It was Hippasus, one of Pythagoras' students, who proved $\sqrt{2}$ to be irrational. Later Theodorus proved $\sqrt{5}$, $\sqrt{6}$ et cetera to be irrational.

28 Leon Perry, The Mathmen, page 40

The famous Greek philosopher Plato made no great contribution to mathematics himself. But he created a school in which he trained and directed the works of so many famous mathematicians that he is numbered among the creators of this science.²⁹ In his school, Academy, he had four 'roads' for the students to follow: astronomy, geometry, arithmetic³⁰, and music. The only very significant thing he did was to show the uniqueness of the Platonic figures; that is to prove that the tetrahedron (4 sides), the hexahedron (6 sides), the octahedron (8 sides), the dodecahedron (12 sides), and the icosahedron (20 sides) are the only possible regular polyhedrons (shapes).

The next figure to enter the scene of Greek mathematics was Eudoxus. He was originally a student of Plato. He did a large amount of work on the theory of proportions (fractions).³¹ His theory of concentric circles, which was an elaborate explanation as to the movement of planets with the earth in the center, lasted many thousand years. Another work of his was the development of a geometric system of irrational numbers.³² He also did work which would become the predecessor to modern integral calculus³³

He (Aristotle) had no special field. His knowledge was universal, and he wrote about everything.

29 Mathematics, History, Encyclopedia Britannica, volume 11, page 645

30 Arithmetic actually meant advanced number theory.

31 Leon Perry, The Mathematics, page 42

32 "Mathematics, History, Encyclopedia Britannica, volume 11, page 641;U

33 Michal Moffatt, The ages of Mathematics vol 1, page 80

Very little is known about the background of Euclid. It is, however, known that he taught in an Egyptian university called Museum. The library of Alexandria, which served Museum, is said to have had 600,000 papyrus rolls!

-Archimedes, the son of an astronomer, was the greatest scientist and mathematician of ancient times, and his equal did not arise until Isaac Newton, two thousand years later."39 Archimedes went to Museum. Despite his many inventions, he was more interested in pure mathematics. A remarkable feat of his was solving problems of differential calculus. He invented the water screw, pulleys, and levers and formulated the laws of buoyancy. He also worked on π decimals and invented basic limits. Some of his major works are: The sand reckoner, which demonstrates that any number can be mathematically expressed; The cattle problem, a challenge to a rival, Apollonius, it has 8 variables with 8 equations with an answer billions of trillions of digits long; The law of the lever, concerning the behavior of levers; On floating bodies, set down laws of buoyancy. He once said "give me another earth to stand on and I shall lift this one" in reference to the powers of levers and pulleys. He also designed remarkable weapons including catapult to hurl 1-ton and 10 pound stones, cranes which could lift and throw ships, and lenses to ignite ships miles away!

39 Isaac Asimov, Asimov on Numbers, page 173

Starting in the 17th century a new wave of mathematical thought developed. Algebra was further developed and new fields invented. Some of the greatest men lived within 200 years of each other during this period. Also new light fell upon previous questions, which were unanswered.

One of the largest unsolved problems was Euclid's parallel postulate. It was unproved by the other axioms up to the 17th century. In 1733 Girolamo Saccheri unsuccessfully tried to prove it by 'reductio ad absurdum' or indirectly. The Russian Nikolai Ivanovich in 1829 and the Hungarian Johann Bolyai in 1832, unknown to each other independently discovered a non-Euclidean geometry. They followed similar lines as Saccheri but asserted that no contradiction could be found.

"It was during the 17th century that John Napier revealed his invention of logarithms. That Galileo Galilei founded the mathematics of dynamics. That Johannes Kepler induced his laws of planetary motion. That Gerolamo Desargues and Blaise Pascal formulated projective geometry. That Pierre de Fermat laid the foundations of modern number theory. And that Pascal, Fermat, and Christiaan Huygens made distinguished contributions to the theory of probability."

"The development of analysis in the 17th century by the mathematicians Pierre de Fermat, Rene Descartes, and Isaac Newton soon left behind classical methods and problems, and an enormous wealth of new discoveries revealed an interaction between theoretical mathematics and all branches of physics and astronomy."

39 "Mathematics", Encyclopedia Americana, volume 17, page 396

40 Ibid

41 "Mathematics, History", Encyclopedia Britannica, volume-II, page 648

42 Ibid

The 19th century had an enormous quantity of new methods which were contradictions of old beliefs.

In 1843, after years of cogitation, the Irish mathematician William Rowan Hamilton was led to invent his quaternion algebra in which the commutative law of multiplication does not hold. 42

In 1844 the German mathematician Hermann Günther Grassman published the first edition of his remarkable Ausdehnungslehre, in which he developed classes of algebras of much greater generality than Hamilton's quaternion algebra. By weakening or deleting various of the laws of common algebra, or by replacing some of the laws by others that are also consistent with the remaining ones, an enormous variety of algebraic structures can be created. 43

Don't use just a line mark when you indicate a single space. They mean the same.

Hamilton and Grassman opened the world to abstract algebra. A mathematician can use any set of consistent axioms he chooses.

"There has never been a man like Newton, and there never will be one like him. Not Einstein, not Archimedes, not Galileo, not Plank, not anybody else measured up to near his stature.,,44 Newton in addition to formulating the laws of gravity invented differential and integral calculus. He developed systems to solve many problems which could not be solved until he solved them. Newton developed an excellent system of limits.

42 "Mathematics", Encyclopedia Americana, volume 17 page 400

43 Ibid

44 Petr Beckmann, A History of ?f, page 137

This paper just sort of dies here. How about a conclusion telling of the tremendous men you have covered. Make a statement. Make the reader feel the paper was worth reading! Most importantly - Conclude. It's as if you

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